

Simulation of Heme Using DFT + U: A Step toward Accurate Spin-State Energetics

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We investigate the DFT + U approach as a viable solution to describe the low-lying states of ligated and unligated iron heme complexes. Besides their central role in organometallic chemistry, these compounds represent a paradigmatic case where LDA, GGA, and common hybrid functionals fail to reproduce the experimental magnetic splittings. In particular, the imidazole pentacoordinated heme is incorrectly described as a triplet by all usual DFT flavors. In this study, we show that a U parameter close to 4 eV leads to spin transitions and molecular geometries in quantitative agreement with experiments and that DFT + U represents an appealing tool in the description of iron porphyrin complexes, at a much reduced cost compared to correlated quantum-chemistry methods. The possibility of obtaining the U parameter from first principles is explored through a self-consistent linear-response formulation. We find that this approach, which proved to be successful in other iron systems, produces in this case some overestimation with respect to the optimal values of U.

I. Introduction

Enzymatic sites containing transition metals are among the most relevant biophysical systems currently studied using first-principles quantum mechanical approaches. The application of such tools, however, is often severely limited as a consequence of the inability of conventional electronic structure methods—such as Hartree–Fock or density-functional theory—to provide a qualitatively correct description of the spin-state energetics of the metal center. Iron porphyrins, which constitute the prosthetic group of the ubiquitous heme proteins, are a paradigmatic example where the aforementioned approaches cannot be relied upon to predict the ground-state multiplicity of the system.

The spin state of iron porphyrins, as much as the spin state of any transition metal complex, is determined by the coordination symmetry and the nature of the ligands. The three lowest accessible spin states (a singlet, a triplet, and a quintuplet if the number of electrons is even, or a doublet, a quartet, and a sextet if it is odd) are conventionally referred to as low, intermediate, and high spin. In unligated porphyrins, the metal is coordinated to four in-plane nitrogen atoms, and experimental studies on model compounds, namely, on Fe(II) tetraphenylporphine (FeTPP), indicate for this coordination mode a triplet ground state.^{1–5} Additional axial ligands produce alternative multiplicities: imidazole gives rise to high-spin hemes, while strong ligand fields as that of diatomic molecules like CO, NO, or O₂ favor low-spin configurations.⁶ Six-coordinated hemes, with two axial ligands, usually exhibit a low-spin state unless the ligand field is extremely weak. Figure 1 depicts a schematic view of the d-state energy levels in three distinctive coordination environments.

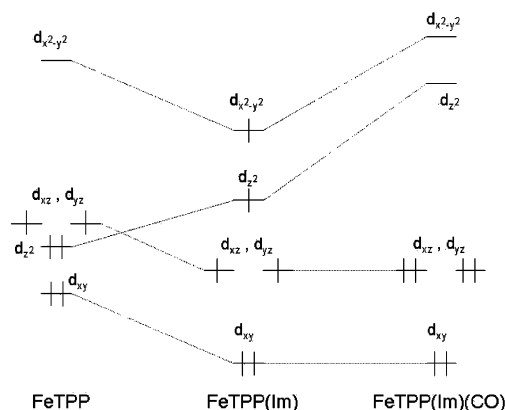


Figure 1. Schematic representation of the d-orbital energy levels for FeTPP. From left to right: free (four-coordinated), ligated to imidazole (five-coordinated), and ligated to imidazole plus CO (six-coordinated).

Even though first-principles approaches, specifically Hartree–Fock (HF) and density-functional theory (DFT), greatly contributed to the interpretation and understanding of the functional aspects of the active site of heme proteins at the molecular level, attempts to predict the ground-state multiplicity of these systems soon made it apparent that an accurate description of the electronic structure might require more sophisticated techniques. This fact can be tracked down to the spin-transition energies provided by HF and DFT for isolated iron atoms and ions or different iron compounds, where it has been systematically observed that HF favors high-spin electronic configurations while DFT exhibits a preference for low-spin states.^{7–12} Such biases are similarly manifested in heme complexes: Table 1 summarizes this trend in five- and six-coordinated iron porphyrins (FeP).

For the past decade, DFT has been the first method of choice to perform electronic structure calculations of biological models, and in particular of heme systems. In this context, one of the

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TABLE 1: Experimental and Calculated Electronic Ground States of Five and Six-Coordinated Iron Porphines (FeP), with the Following Axial Ligands: O₂, CO, Imidazole (Im), and Chloride

	six-coordinated		five-coordinated		
	FeP(Im)(O ₂)	FeP(Im)(CO)	FeP(CO)	FeP(Im)	FeP(Cl)
experimental	singlet	singlet	singlet	quintuplet	sextet
Hartree–Fock	quintuplet	quintuplet	quintuplet	quintuplet	sextet
DFT-GGA	singlet	singlet	singlet	triplet	quartet
B3LYP	singlet	singlet	singlet	triplet	quartet/sextet

most crucial failures of common exchange-correlation functionals has been detected in the deoxygenated active site of hemoglobin and myoglobin (Table 1). The earliest study reporting this flaw is due to Rovira et al.,¹³ who obtained for the five-coordinated model Fe^{II}P(Im) (axial ligand: imidazole) a triplet state 6.5 kcal/mol below the quintuplet, which is the experimental ground state of the system. After this work, a few others followed which also observed this inversion using B3LYP or different pure GGA functionals.^{7,14} Liao and Scheiner claimed to have found a quintuplet ground state for this five-coordinated compound employing a DFT-GGA functional.¹⁵ In their calculations, however, electronic symmetry constraints were imposed. To the best of our knowledge, DFT functionals yield for Fe^{II}P(Im) a triplet ground state in the absence of symmetry constraints. In an effort to quantify the errors in the DFT estimates of spin-transition energies, Ghosh and Taylor resorted to highly correlated techniques such as CASPT2 and CCSD(T) to explore the iron(III) porphyrin chloride.⁸ This is another example of a high-spin five-coordinated heme complex for which DFT predicts a quartet favored over the sextet, in this case by around 7 kcal/mol. B3LYP, on the other hand, finds about the same energy for both spin configurations. The more accurate approaches CASPT2 and CCSD(T) agree in yielding a sextet ground state, 16 kcal/mol below the quartet.⁸ A latter work by these authors shows the same low-spin bias in B3LYP for the iron(IV) porphyrin difluoride.⁹ It is worth noting here that even CASPT2, employed with the moderate active spaces currently affordable, has been found to be fallible in the estimation of these elusive spin states. Inaccuracies have been reported in the prediction of the electronic ground states of the isolated iron porphyrin¹⁶ and of the oxyheme.¹⁷

It is possible to find a rationale for the biases in DFT and HF, considering the balance between the computed electronic exchange and correlation energies. In a simplified picture, the (negative) exchange energy is contributed by like-spin electron pairs, while electronic correlation arises from the interaction between electrons regardless of their spin. A method which includes the exchange and neglects the correlation, as HF does, will favor high multiplicities by maximizing the number of electrons with the same spin. To the contrary, experience shows that the combination of the exchange and correlation terms in pure DFT pushes the balance toward low-spin configurations.¹² Attempts to improve the spin-state energetics description of density functionals have mostly been based on hybrid Hartree–Fock/DFT schemes,^{7,10,12} which combine the exchange of HF with the exchange and correlation obtained from DFT in proportions obeying empirical considerations. This approach, however, has given no universal functional capable of providing accurate splittings in every case. In general, those functionals offering a good description of the high-spin species fail when tried out on low-spin complexes, and vice versa.⁷ Among them, B3LYP is seemingly the one with the best average performance up to now, yet exhibiting serious inaccuracies in the five-coordinated models already discussed.

In the present study, we propose the DFT + U approach as an alternative to the standard ab initio techniques for a reliable description of the low-lying states of iron heme complexes. The LDA + U or GGA + U method (more generally denoted as DFT + U) was originally designed within the density-functional theory framework for the treatment of strongly correlated materials.^{18–22} Only very recently researchers have started to apply it to molecular, or mixed solid–molecular systems, with extremely promising results.^{23–27} This approach corrects the tendency to overhybridize and delocalize electronic orbitals, ultimately originating in the presence of self-interactions in the exchange-correlation functionals, by introducing a term that penalizes fractional occupancies. We note in passing that in our present implementation we explore the possibility of a U that is not a best-fit parameter but an intrinsic, ab initio linear-response property of the system chosen. However, this approach does not prove to be totally satisfactory in the case of low-spin complexes, for which it leads to values of U lying 1 or 2 eV above the optimal ones. We show that, with the inclusion of a single parameter, DFT + U recovers the correct multiplicities of the five-coordinated models where DFT and hybrid methodologies are in disagreement with more elaborated techniques or experimental data. Moreover, there is no impairment with respect to GGA functionals in those cases for which DFT displayed the right behavior. Calculations of ligand exchange thermodynamics, spin transitions, and other properties point to GGA + U as an appealing tool to overcome the limitations entailed by the use of DFT in the description of bioinorganic complexes, at a computational expense much lower than that demanded by highly correlated quantum-chemistry methods.

II. Methodology

A. General Framework. All calculations reported in this work have been performed with the public domain PWSCF and CP codes included in the Quantum-Espresso distribution,²⁸ based on density-functional theory, periodic-boundary conditions, plane-wave basis sets, and pseudopotentials to represent the ion–electron interactions. The PBE exchange–correlation functional²⁹ has been used in combination with Vanderbilt ultrasoft pseudopotentials,³⁰ with the Kohn–Sham orbitals and charge density expanded in plane waves up to a kinetic energy cutoff of 25 and 200 Ry, respectively.

B. The DFT + U Approach. The present implementation of DFT + U stems from the early contributions by Anisimov and others,^{18–22} who proposed to correct the failures of the LDA functional in dealing with the strongly localized d or f electrons of transition metal ions. An on-site correction was thus constructed to account for strong electronic correlations poorly described within the local-density or generalized-gradient approximations and formulated as follows:

$$E_{\text{DFT+U}}[\rho(\mathbf{r})] = E_{\text{DFT}}[\rho(\mathbf{r})] + E_{\text{U}}[\{n_{mm'}^{\sigma}\}] = E_{\text{DFT}}[\rho(\mathbf{r})] + E_{\text{HUB}}[\{n_{mm'}^{\sigma}\}] - E_{\text{DC}}[\{n^{\sigma}\}] \quad (1)$$

where $\rho(\mathbf{r})$ is the electronic density, $n_{mm'}^{I\sigma}$ are generalized atomic orbital occupations with spin σ associated to the I atom, and $n^{I\sigma}$ is the sum of the occupations corresponding to all eigenstates, $\sum_m n_{mm'}^{I\sigma} E_{\text{DFT}}[\rho(\mathbf{r})]$ is the standard LDA or GGA energy functional, and $E_{\text{HUB}}[\{n_{mm'}^{I\sigma}\}]$ represents the “correct” on-site correlation energy. Since $E_{\text{DFT}}[\rho(\mathbf{r})]$ already contains an approximate correlation contribution, a term intended to model such a contribution, $E_{\text{DC}}[\{n^{I\sigma}\}]$, must be subtracted to avoid double counting.

In this work, we resort to the rotationally invariant formulation of DFT + U introduced by Liechtenstein et al.²¹ and later simplified by Dudarev and his co-workers,²² in which the nonsphericity of the electronic interactions and the differences among the interactions in like-spin and unlike-spin channels are neglected. With these assumptions, the correction to the energy functional can be written

$$E_{\text{U}}[\{n_{mm'}^{I\sigma}\}] = \frac{U}{2} \sum_I \sum_{m,\sigma} [n_{mm}^{I\sigma} - \sum_{m'} n_{mm'}^{I\sigma} n_{m'm}^{I\sigma}] = \frac{U}{2} \sum_{I,\sigma} \text{Tr}[n^{I\sigma}(1 - n^{I\sigma})] \quad (2)$$

where U is the Hubbard parameter describing on-site correlations. In principle, different definitions for the occupation matrix are possible, which in turn will determine different values for U . In this case we define

$$n_{mm'}^{I\sigma} = \sum_{\nu} f_{\nu} \langle \psi_{\nu}^{\sigma} | \phi_m^I \rangle \langle \phi_{m'}^I | \psi_{\nu}^{\sigma} \rangle \quad (3)$$

with f_{ν} being the weight of the electronic state ν , ϕ_m^I the valence atomic orbital $|lm\rangle$ of atom I , and ψ_{ν}^{σ} the one-electron wavefunction corresponding to the state ν with spin σ . The diagonalization of the occupation matrices leads to the following expression for the energy correction:

$$E_{\text{U}}[\{n_{mm'}^{I\sigma}\}] = \frac{U}{2} \sum_{I,\sigma} \sum_i \lambda_i^{I\sigma} (1 - \lambda_i^{I\sigma}) \quad (4)$$

Equation 4 clearly reflects the nature of the correction, which imposes a penalty (mediated by U) for fractional occupations, thus favoring either fully occupied or empty orbitals ($\lambda \approx 1$ and $\lambda \approx 0$, respectively). We note that, under this definition, U corresponds to the difference $U - J$, as utilized by Anisimov and other researchers.^{20–22} For example, the adoption of $U = 4$ eV in the present calculations is comparable to a U value of 5 eV in combination with a J value of 1 eV in the work of Rollmann.²⁴ Whereas in recent applications U is considered a fitting parameter,^{24–27} here we obtain it from the spurious curvature of the DFT energy as a function of the occupations. As shown by Cococcioni and de Gironcoli,³¹ the value of U can be estimated as the difference between the screened and bare second derivative of the energy with respect to the occupations:

$$U = \frac{\partial^2 E_{\text{DFT}}}{\partial (n^I)^2} - \frac{\partial^2 E_{\text{DFT}}^{\circ}}{\partial (n^I)^2} \quad (5)$$

In particular, we are interested in the self-consistent U , which we will call U_{sc} , originating from the curvature of the DFT + U ground state.²³ To compute U_{sc} , a few linear-response calculations must be performed at a finite U_{in} value, each one yielding a corresponding U_{out} value. It can be shown that there

is a linear dependence between U_{in} and U_{out} , from which U_{sc} can be extrapolated:²³

$$U_{\text{out}} = \frac{\partial^2 E_{\text{quad}}}{\partial (n^I)^2} = U_{\text{sc}} - \frac{U_{\text{in}}}{m} \quad (6)$$

E_{quad} groups all electronic terms within the DFT + U functional that have quadratic dependence on the occupations, whereas m can be interpreted as an effective degeneracy of the orbitals whose population is perturbed. This procedure, which allowed us to attain an improved description of the multiplet splittings and bonding in Fe dimers and FeO related species,²³ is the one adopted here to calculate a self-consistent U parameter for the iron porphyrin system. Another criterion has also been explored, requesting that a linear-response calculation at a finite U value returns this same value of U at the output, for example, $U_{\text{in}} = U_{\text{out}}$. The parameter fulfilling this criterion will be hereafter denoted U'_{sc} . This second criterion is not as appealing as the first one, since U_{sc} seems to be the “right definition” for self-consistency.

III. Results and Discussion

In this section, DFT + U results are presented on four heme complexes: $\text{Fe}^{\text{II}}\text{P}(\text{Im})$, $\text{Fe}^{\text{III}}\text{P}(\text{Cl})$, $\text{Fe}^{\text{II}}\text{P}(\text{Im})(\text{O}_2)$, and $\text{Fe}^{\text{II}}\text{P}(\text{CO})$. In the first two cases, DFT calculations fail to predict the high-spin nature of the system. The other two are examples of low-spin hemes whose electronic and geometrical properties are, in principle, correctly captured by standard density-functional simulations. These four case studies were chosen for their respective relevance in bioinorganic chemistry, and to illustrate the performance of the DFT + U method on heme models exhibiting a variety of coordination modes and multiplicities.

A. Five-Coordinated Heme–Imidazole Complex. The $\text{Fe}^{\text{II}}\text{P}(\text{Im})$ system, depicted in Figure 2, has been the target of numerous computational studies, inasmuch as it appears as the natural model to represent the unbound active site of several heme proteins, in particular hemoglobin and myoglobin.^{7,13–15,32} As mentioned above, the ground state of this compound has been experimentally characterized as a quintuplet ($S = 2$), whereas DFT calculations yield a triplet ground state ($S = 1$). In Table 2, the energetic separations between the low-lying spin states resulting from DFT and DFT + U are compared. According to PBE, the triplet is around 8 kcal/mol more stable than the quintuplet or the singlet; similar gaps are obtained with the BP86 exchange–correlation functional.¹³ On the other hand, the transition energy between the triplet and the quintuplet is reduced to nearly 2 kcal/mol if B3LYP is used.¹⁴ As previously noted, the introduction of the HF exchange in the DFT functional stabilizes high multiplicity states, but in this case, B3LYP is still unable to provide the right splittings.

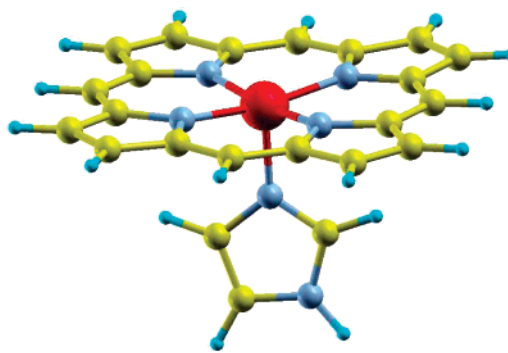
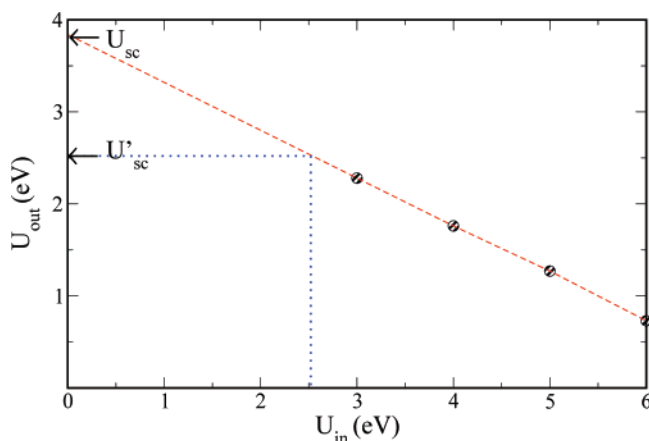
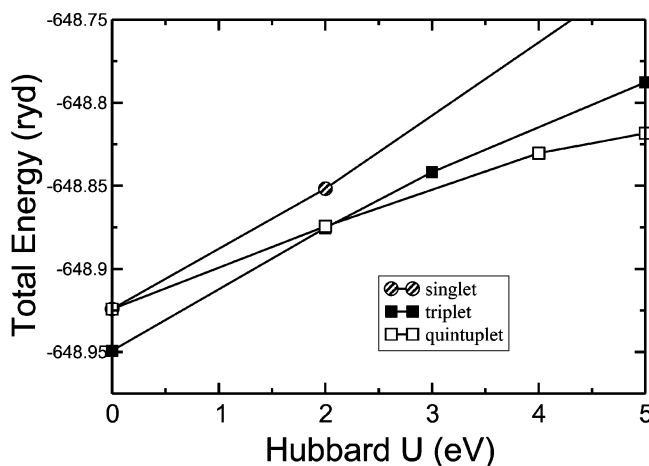


Figure 2. Structure of the five-coordinated $\text{Fe}^{\text{II}}\text{P}(\text{Im})$ complex.

TABLE 2: Spin-Transition Energies (kcal/mol) for the Low-Lying Spin States of Fe^{II}P(Im) Calculated with Several Density Functionals and with DFT + U, Using $U_{sc} = 3.9$ eV

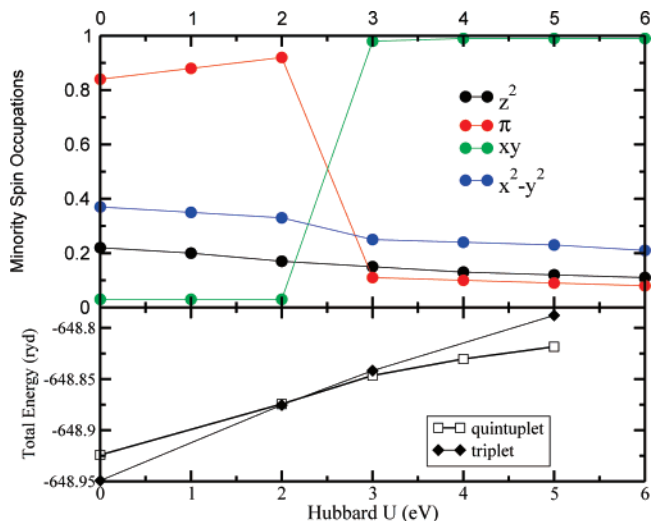
	singlet	triplet	quintuplet
DFT			
PBE ^a	7.8	0.0	7.9
BP86 ^b	8.3	0.0	6.5
B3LYP ^c	5.8	0.0	1.9
DFT + U			
	20.9	4.9	0.0

^a Pseudopotential calculations with plane-wave basis sets. ^b Pseudopotential calculations with plane-wave basis sets from ref 13. ^c Gaussian calculations (VTZ basis) from ref 14.

**Figure 3.** Linear-response calculation of the U parameter on the quintuplet state of the Fe^{II}P(Im) complex.**Figure 4.** Total energy of the low-lying spin states of the Fe^{II}P(Im) complex as a function of the U parameter.

The formalism summarized in eq 6 gives for this system a U_{sc} value of 3.9 eV and a U'_{sc} value of 2.5 eV (Figure 3). If any of these values are adopted, DFT + U restores the experimental ordering of the spin states. The total energies of the low-lying states as a function of U are depicted in Figure 4. The increase of the U parameter equalizes the triplet and quintuplet energies, producing a spin crossover at $U \approx 2$ eV. At higher values of U , the quintuplet remains the ground state.

Figure 5 highlights the effect of U on the electronic symmetry of the d states in the heme porphyrin. Spin occupations were computed by projecting the electronic wavefunctions on the atomic orbitals of the iron, as prescribed by eq 3. The upper panel of Figure 5 represents the occupations of the minority spin manifold in the quintuplet state. Iron(II) is a d^6 ion, and the quintuplet bears four unpaired electrons; therefore, the sum

**Figure 5.** Occupations of the minority spin manifold in the quintuplet state of the Fe^{II}P(Im) complex. The lower panel shows the total energy of the two lowest accessible spin states as a function of U .**TABLE 3: Selected Experimental and Optimized Structural Parameters (Å) in Fe^{II}P(Im)**

	d_{Fe-p}	Fe–N _{porph}	Fe–N _{imi}
experimental ^a	0.42	2.09	2.16
DFT (PBE)	0.29	2.07	2.13
DFT + U ($U_{sc} = 3.9$ eV)	0.43	2.11	2.19

^a Data for Fe^{II}TPP(2Me-Im) from ref 35.

of the occupations on the minority spin channel should be around 1. It is not exactly 1 because the eigenstates of the complex do not correspond to pure d atomic orbitals but are instead strongly hybridized. However, it is possible to assign the electronic configuration of the system in terms of d atomic orbitals, depending on whether the occupations are close to 0 or 1. Note that, by convention, the nitrogen atoms of the porphyrin ring are placed on the xy plane, with the x and y axes oriented along the Fe–N bonds. Pure DFT ($U = 0$) and DFT + U with $U < 2$ eV converge to the $(d_z^2)^1(d_{xy})^1(d_{xz})^3(d_{x^2-y^2})^1$ state.³³ This is the same configuration as reported by Spiro and co-workers from B3LYP simulations.¹⁴ The increase of U above 2 eV stabilizes the $(d_z^2)^1(d_{xy})^2(d_{xz})^2(d_{x^2-y^2})^1$ state, which is the one experimentally assigned to Fe^{II}P(Im).⁶ Interestingly enough, this change in configuration is associated with an inversion in the relative energy of the triplet and the quintuplet, which now becomes the ground state.

The examination of the optimized geometry at a finite U value of 3.9 eV shows an agreement with the experimental data at least as good as pure DFT does. The structural parameters most affected by the U correction are those in the vicinity of the metal center, presented in Table 3. The out-of-plane displacement of the iron, d_{Fe-p} , is the distance of the iron to the average plane defined by the four nitrogen atoms of the porphyrin ring. The interplay between spin state and d_{Fe-p} , often involved in the dynamics of the heme protein—as for example in the allosteric mechanism of hemoglobin³⁴—has been characterized experimentally⁶ and theoretically.^{13,14} Table 3 contrasts this and other optimized structural parameters with the experimental data available for the synthetic model compound Fe^{II}TPP(2-MeIm) (TPP, tetraphenyl porphine; 2-MeIm, 2-methyl imidazole).³⁵ Figure 6 shows the dependence of d_{Fe-p} on U , with the shaded part of the graph indicating the experimental region.

In summary, the U term favors the stabilization of the $(d_{xy})^2$ configuration—deemed the experimental ground state of the

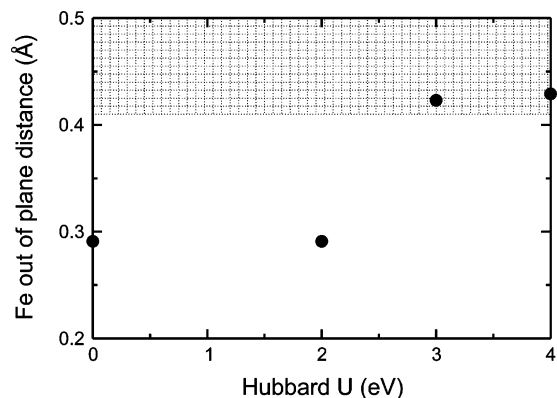


Figure 6. Distance of the iron to the average plane defined by the four nitrogen atoms of the porphyrin ring as a function of U in $\text{Fe}^{\text{II}}\text{P}(\text{Im})$. The shaded area encompasses the experimental region.

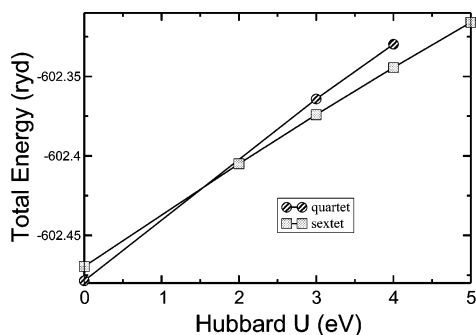


Figure 7. Total energy of the lowest accessible spin states of the $\text{Fe}^{\text{II}}\text{P}(\text{Cl})$ complex as a function of the U parameter.

TABLE 4: Spin-Transition Energies (kcal/mol) for the Low-Lying Spin States of $\text{Fe}^{\text{II}}\text{P}(\text{Cl})$ Calculated with Highly Correlated Methods and Density-Functional Theory, Including DFT + U ($U_{\text{sc}} = 4.0$ eV)

	quartet	sextet
CASPT2 ^a	19.6	0.0
RCCSD(T) ^b	16.1	0.0
DFT-PBE	0.0	5.6
DFT-PW91 ^a	0.0	8.1
DFT + U	9.2	0.0

^a Reference 37. ^b Calculations on a simplified model, ref 38.

complex—rendering this (quintuplet) state the lowest in energy, as can be seen in the lower panel of Figure 5.

B. Iron(III) Porphyrinato Chloride. The low-lying accessible electronic states for the pentacoordinate $\text{Fe}^{\text{III}}\text{P}(\text{Cl})$ complex, with five d electrons, are the sextet, the quartet, and the doublet ($S = 5/2$, $S = 3/2$, and $S = 1/2$, respectively). Similarly to the situation discussed in the previous section, DFT cannot reproduce the high-spin character of the system, which has been established experimentally.³⁶ Using a battery of ab initio methods, Ghosh and collaborators have explored this complex in depth.^{8,37,38} They found that, while the PW91 exchange-correlation functional yields a quartet state 8.1 kcal/mol more stable than the sextet, B3LYP provides nearly identical energies for both configurations. Higher-level CASPT2 calculations and CCSD(T) simulations on a smaller model system are consistent with experiments, placing the sextet almost 20 kcal/mol below the quartet.^{37,38} These results are summarized in Table 4.

Figure 7 shows that, as seen in $\text{Fe}^{\text{II}}\text{P}(\text{Im})$, the U term stabilizes the highest multiplet in $\text{Fe}^{\text{III}}\text{P}(\text{Cl})$. A spin inversion is verified at $U \approx 1.5$ eV, rendering the sextet as the ground state. A value of U_{sc} equal to 4.0 eV is obtained, which leads to a sextet–quartet transition energy of 9.2 kcal/mol. Table 4 makes evident

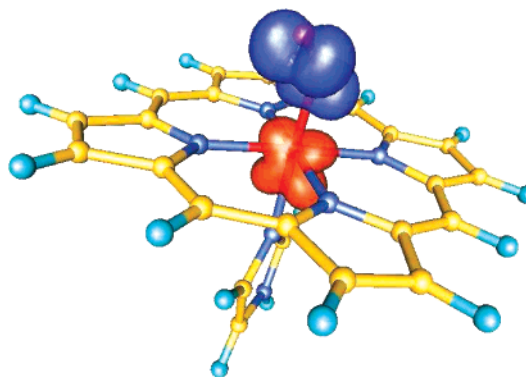


Figure 8. Spin density in $\text{Fe}^{\text{II}}\text{P}(\text{Im})(\text{O}_2)$ corresponding to an open-shell singlet, calculated with DFT + U . Lobes localized on the iron and on the O_2 represent unpaired electron density of opposite spin.

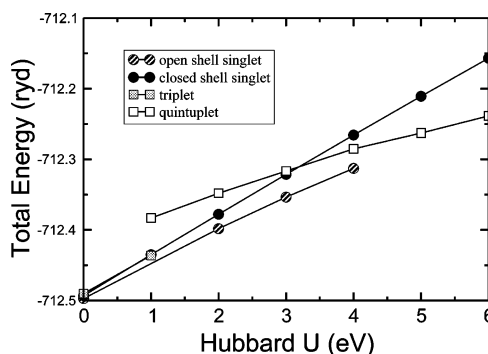


Figure 9. Total energy of the low-lying spin states of $\text{Fe}^{\text{II}}\text{P}(\text{Im})(\text{O}_2)$ as a function of the U parameter.

the poor performance of density functionals to describe multiplet splittings in transition metals, capable of errors in the order of tens of kilocalories per mole. Despite its quantitative disagreement with the highly correlated methods (whose ultimate accuracy is, on the other hand, difficult to assess in this case), DFT + U succeeds in recovering the ordering of the spin states.

C. Six-Coordinated Oxyheme Model. The $\text{Fe}^{\text{II}}\text{P}(\text{Im})(\text{O}_2)$ system has been long identified as low spin in native proteins and in synthetic compounds.⁶ Its importance as the oxygenated model of hemoglobin and myoglobin is reflected in the literature, which, aside from the experimental work, offers many computational studies addressing the electronic and structural aspects of the complex.^{13,32,39} The low-spin nature of six-coordinated iron porphyrins is in general correctly described by DFT, consequently with its trend to destabilize high multiplicity states. In the particular case of $\text{Fe}^{\text{II}}\text{P}(\text{Im})(\text{O}_2)$, calculations with different functionals, including B3LYP, indicate a singlet ground state of open-shell character.^{7,13} While the total spin of the molecule is zero, DFT calculations reveal partial spin densities localized on the d orbitals of Fe and the π^* orbitals of O_2 , integrating approximately to +1 and −1, corresponding to two unpaired electrons of opposite spin.^{7,13} This open-shell singlet (oss) state can be interpreted as the result of an antiferromagnetic coupling between $\text{Fe}^{\text{II}}\text{P}(\text{Im})$ ($S = 2$) and O_2 ($S = 1$), each retaining part of its magnetic character upon binding.

DFT + U supports this picture: Figure 8 depicts the spin density, $\rho_{\text{spin}}(\mathbf{r}) = \rho_{\alpha}(\mathbf{r}) - \rho_{\beta}(\mathbf{r})$, computed at a finite U value of 4 eV. This figure is qualitatively equivalent to the one reported by Rovira and co-workers using pure DFT.¹³ The impact of the U parameter on the total energies of the lowest accessible spin states is plotted in Figure 9. The progressive increment of U further stabilizes the oss with respect to the closed-shell singlet and the triplet. On the other hand, the gap

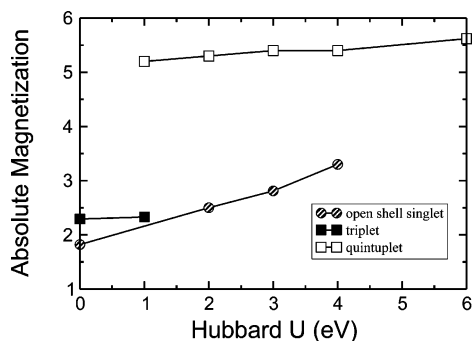


Figure 10. Absolute magnetization of the low-lying spin states of $\text{Fe}^{\text{II}}\text{P}(\text{Im})(\text{O}_2)$ as a function of the U parameter.

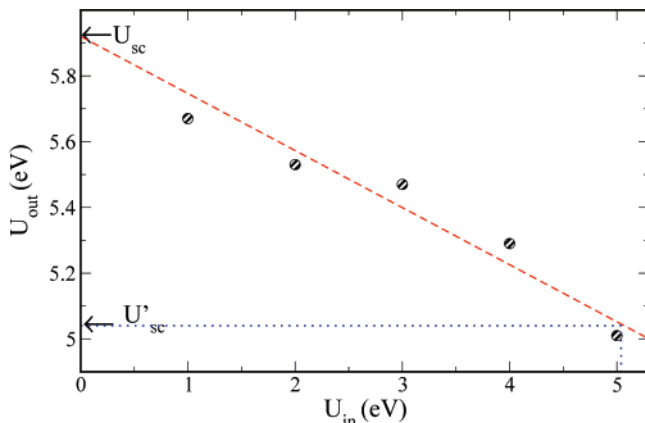


Figure 11. Linear-response calculation of the U parameter on the open-shell singlet state of the $\text{Fe}^{\text{II}}\text{P}(\text{Im})(\text{O}_2)$ complex.

between the oss and the quintuplet is reduced, but the raise in U beyond 4 eV produces the dissociation of the Fe–O bond in the oss before a spin crossing between these two states is observed. The effect of the on-site correction can also be examined through the absolute magnetization of the molecule, defined as $\int |\rho_\alpha(\mathbf{r}) - \rho_\beta(\mathbf{r})| d\mathbf{r}$, a measure of the unpaired electron density in the system. Figure 10 illustrates how the on-site correlation affects the distribution of the unpaired electron density, reinforcing, in particular, the antiferromagnetic character of the oss. The net effect of U is seemingly to thwart the coupling of the unpaired electrons of the molecular oxygen and the porphyrin, stabilizing the separate species, which is reflected in the elongation of the Fe–O distance discussed below. The absolute magnetization augments from 1.8 at $U = 0$ to 3.2 at $U = 4$ eV, a value placed halfway from that corresponding to the unbound system, comprised of a triplet plus a quintuplet ($S = 1 + 2$).

Extrapolation to the y axis in the $U_{\text{in}}-U_{\text{out}}$ plot (Figure 11) yields a U_{sc} value of 5.9 eV, sensibly higher than in the previous examples. This value induces the rupture of the Fe–O bond, and in consequence, it is not possible to obtain a relaxed, bound complex associated with this U_{sc} value. As seen in Figure 12, where the Fe–O distance is plotted as a function of U , the increase in the on-site correction provokes the elongation of the bond, eventually leading to the dissociation of the complex. The shaded area in the graph encloses the range of experimental Fe–O lengths observed in different model compounds. It is important to emphasize that such values correspond to synthetic or natural compounds in which the O_2 molecule is stabilized in the axial position by virtue of a second interaction on the distal side, namely, a hydrogen bond or some kind of trapping or cage effect. In the absence of a distal cavity, oxygenation of iron(II) porphyrins under ordinary conditions has been rarely ob-

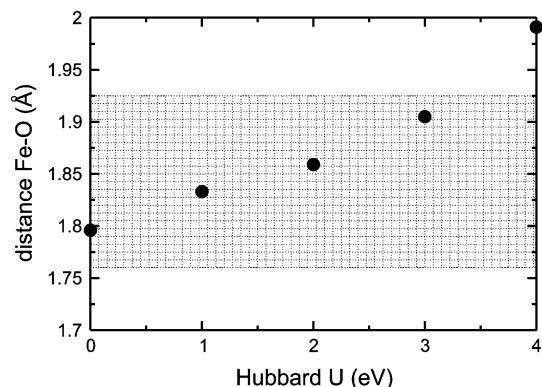


Figure 12. Fe–O bond length as a function of the U parameter in $\text{Fe}^{\text{II}}\text{P}(\text{Im})(\text{O}_2)$. The shaded area encompasses the experimental region.

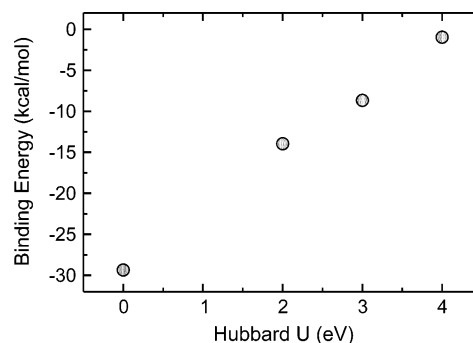


Figure 13. Energy of O_2 binding to $\text{Fe}^{\text{II}}\text{P}(\text{Im})$ as a function of the U parameter.

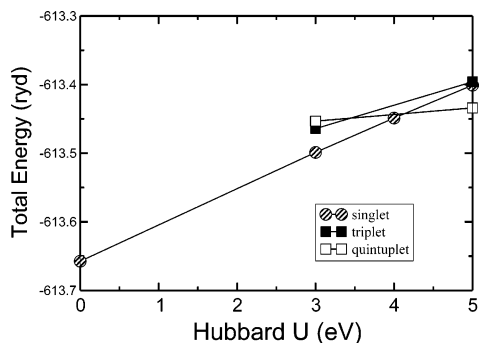
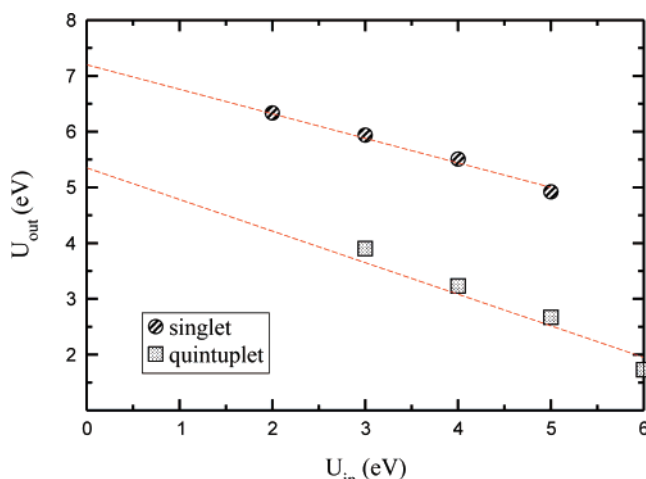
served.^{40,41} Pure DFT and B3LYP systematically overestimate the binding of O_2 , giving, for the free heme, energies a range of 15–25 kcal/mol,^{13,32} in direct contradiction with the experimental difficulty to isolate the oxygenated species. While part of this error is associated with the underestimation of the total energy of the quintuplet, such overbinding represents a major problem in the application of DFT to the calculation of affinity constants. Figure 13 shows that DFT + U provides a more realistic oxygen affinity, with the binding energy decreasing from 28 kcal/mol at $U = 0$ to around 1 kcal/mol at $U = 4$ eV. In the present case, U'_{sc} turns out to be 1 eV lower than U_{sc} (Figure 11). A U value of 5.8 eV, as obtained from eq 6, is too high to represent the thermodynamic and geometrical properties of the oxygenated complex. This is evincing a positive bias in the linear-response approach, which will be manifested also in other low-spin systems. We will come back to this issue later in the next sections.

D. Five-Coordinated Carboxyheme Model. As the last case study, we will briefly discuss the $\text{Fe}^{\text{II}}\text{P}(\text{CO})$ complex. Five- and six-coordinated carboxyhemes are low-spin systems whose electronic and geometrical features are well reproduced by DFT, notwithstanding the overestimation of the CO binding energy, similarly to what is found with O_2 . The motivation to include the carboxylated complex in this study is therefore to assess the behavior of DFT + U in comparison with standard density-functional theory, in particular to examine if the former produces any detrimental bias in a case where the latter shows already a good performance. Table 5 contains computed and experimental values for a few selected properties of the carboxyheme. The general agreement between the simulations and the X-ray data is benefited from the U term, which not only provides a marginal improvement on the geometrical parameters of the complex but also corrects for the overbinding trend exhibited by pure DFT. At the same time, however, the enhancement of the on-site

TABLE 5: Selected Bond Distances (Å) and CO Binding Energy (kcal/mol) for the Fe^{II}P(CO) Complex Calculated with DFT + U

U (eV)	Fe–C(CO)	Fe–N _{porph}	C–O	ΔE
0.0	1.69	1.99	1.17	−45.3
3.0	1.71	2.00	1.16	−26.1
5.0	1.74	2.01	1.16	−12.0
experimental ^a :	1.77	2.02	1.12	

^a The experimental model is axially coordinated to pyridine, ref 42.

**Figure 14.** Total energy of the low-lying spin states of Fe^{II}P(CO) as a function of the U parameter.**Figure 15.** Linear-response calculation of the U parameter on the singlet and quintuplet states of the Fe^{II}P(CO) complex.

correlations closes the gap between the singlet and the quintuplet, to the extent that for $U \geq 4$ eV the latter becomes the most stable state (Figure 14).

The response of the system to the change in orbital occupations resembles the case of Fe^{II}P(Im)(O₂), yielding for U_{sc} and U'_{sc} values of 7.2 and 5.3 eV, respectively (Figure 15). Since neither experimental nor reliable theoretical estimations of the spin-transition energies are available, we cannot evaluate the magnitude of the error in the self-consistent U obtained with each criteria. In principle, only values below 4 eV are consistent with the experimental singlet state, and thus, it is evident that both U_{sc} and U'_{sc} suffer from some overestimation. Interestingly enough, the self-consistent U parameter calculated in a high-spin configuration of the carboxyheme turns out to be significantly smaller, as depicted also in Figure 15. On the other hand, linear-response calculations on the low-spin state of the Fe^{II}P(Im) system return values of U_{sc} above 6 eV (data not shown). This is indicating that the response of the system depends more on the multiplicity than on the particular geometry, coordination mode, or nature of the ligands.

IV. Conclusions and Final Remarks

The inclusion of on-site correlations via a Hubbard term in DFT rectifies the trend of density functionals to overstabilize low-spin states in iron porphyrins. At variance with DFT, with Hartree–Fock, and with hybrid methods, which successfully describe some of the possible coordination modes of the complex but fail in the rest,⁷ DFT + U is capable of providing the qualitatively correct splittings in low- and high-spin iron porphyrins at the same time, if the proper parameter is adopted. This improvement is also reflected in the geometry optimizations and, more importantly, in more realistic binding energies to diatomic ligands. The question of whether a hybrid functional with the proper exchange and correlation contributions would be capable to recover the spin-state energetics of the full iron porphyrins series is still open. We have addressed this question in a previous article,⁷ with no positive results. In our experience, the combination of the HF exchange with the GGA exchange correlation leads to hybrid methods reflecting either the behavior of pure DFT, overstabilizing low-spin states, or the behavior of Hartree–Fock, favoring high-spin states. To the best of our knowledge, no hybrid has been reported that retains the best of both approaches in the description of iron porphyrins, but a more extensive search is probably needed before giving a definite answer.

The application of the linear-response calculation to low-spin states leads to self-consistent Hubbard parameters 1 or 2 eV above the optimal ones. The linear response of the system appears to be more dependent on the spin state than on the coordination number or the identity of the ligands. In fact, plots of U_{in} versus U_{out} belonging to different complexes in the same spin state exhibit a high similarity. The reason for the overestimation of the Hubbard U in low-spin configurations is not evident. Different extensions to the linear-response approach were explored, including the partition of U into U^α and U^β to discriminate between both spin channels, and even between the five d states. Additionally, a J term to represent separately the on-site exchange was implemented. The modifications described above, however, produced little or no effect on the resulting U_{sc} value. The investigation of other factors which could be responsible for these biases is in progress. In any case, values of U of 4 eV or slightly lower seem to be optimal to reproduce the electronic, thermodynamic, and structural properties of the heme compounds.

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